Beveridgean unemployment gap in Finland*

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Abstract

Following a recently proposed sufficient statistics approach that builds on the socially optimal trade-off between vacancies and unemployment along the empirical Beveridge curve, we study the efficient unemployment rate in Finland and a number of other European countries. On average, the efficient rates of unemployment are found to be below realized unemployment rates and quite stable over time. For Finland, the resulting Beveridgean unemployment gap indicates a chronically slack labour market with large inefficiencies during economic downturns. Unemployment gaps behave countercyclically in other countries as well.

Keywords: Beveridge curve, Sufficient statistics, Labour market, Economic slack

JEL Codes: E24, E32, E6, J63, J64, J68

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1 Introduction

The state of the business cycle is a central policy-relevant question in macroeconomics. Unnecessarily high unemployment is not socially desirable as it creates various social problems along with all the lost economic output. Likewise, labour markets may be too tight in the sense that there is too much demand for scarce labour which may lead to economic inefficiency. Since it is crucial to understand the nature of the current economic situation when designing macroeconomic stabilization policies, a myriad of indicators are frequently used to gauge the current state of the cycle.

In a recent paper, Michaillat and Saez (2021) derive an expression for socially optimal level of unemployment using a sufficient statistics approach. Efficient rate of unemployment is the solution to a planner's problem of maximizing social welfare subject to an empirical relationship between vacancies and unemployment — the Beveridge curve.[1] Optimal solution is found on the Beveridge curve at a point in which the marginal gain to social welfare from additional employment is equal to marginal cost from additional costs of recruitment required to support that level of employment. The solution can be characterized with the help of three sufficient statistics: social cost of unemployment, recruiting cost and the Beveridge elasticity, which is tied to the slope of the Beveridge curve.

This paper applies the approach of Michaillat and Saez (2021) in a European context. Our main focus is on the Finnish labour market. However, we provide results for three peer countries in Sweden, Germany and the Netherlands as well. By looking at results also for these countries we are not only contributing to the literature but also partly validating the empirical approach used for Finland. Since the US and European labour markets have differences between them, e.g. in terms of the level of unemployment, applying the method to multiple different countries at once enables us to evaluate how the estimates might differ in different contexts.

We find that in all of the countries studied labour markets have been on average ineffi-

[1]For a survey article of the Beveridge curve, see Elsby et al. (2015).

ciently slack over time. Whereas both German and Dutch labour markets have seen periods of also too tight labour markets, Finland and Sweden have only barely reached the efficient level at times while for most of the sample labour markets have been too slack. For both of these countries our results suggest an efficient rate of unemployment of around 6% which is higher than in Germany or the Netherlands. Extending the Finnish sample with older vintages of data reveals that Finnish labour markets have been chronically too slack. A large shift of the Beveridge curve during the 1990s depression led to an increase in the efficient unemployment rate.

The formula derived by Michaillat and Saez (2021) is appealingly simple as only knowledge of the empirical Beveridge curve along with social costs associated with vacancies and unemployment are needed to calculate the socially efficient outcome. Beveridgean unemployment gap has links to the theory of efficiency in modern labour-market models such as Mortensen and Pissarides (1994), Pissarides (2000) or Shimer (2005) that build on the search and matching framework and thus feature both vacancies and unemployment which are linked through a Beveridge curve. However, the present analysis does not require the specification of a full model or a matching function in order to analyze the socially efficient outcome.[2]

In a more recent paper Michaillat and Saez (2022) advocate for a even simpler formula for the efficient unemployment rate: $u^* = \sqrt{uv}$. This rate of unemployment, they argue, minimizes the nonproductive use of labour which is well approximated by $u + v$. The simplified formula $u^* = \sqrt{uv}$ is a special case of the formula presented in Michaillat and Saez (2021) with certain parameter values, including a Beveridge elasticity of 1. When the social cost of vacancies and unemployment are equal, unit elasticity of the Beveridge curve implies that nonproductive use labour is minimized whenever labour market tightness v/u is equal to one and there are as many vacancies as there are unemployed. Given our estimates for Beveridge elasticity in a number of European economies, this simpler formula does not seem

[2]The social planner's problem in Michaillat and Saez (2021) generalizes the problem in Hosios (1990).

to work as a universal rule for the efficient rate of unemployment outside the US even if it is a good approximation in that case. We find estimates of Beveridge elasticity to vary between countries and this has implications for country-level estimates of the efficient rate of unemployment.

The paper proceeds as follows. Section 2 presents the sufficient statistics approach and discusses values for different parameters. Section 3 gives an overview of the data used in the analysis while section 4 presents our results. Section 5 concludes.

2 Beveridgean unemployment gap

2.1 Sufficient statistics approach

The Beveridgean unemployment gap is defined by Michaillat and Saez (2021) as the difference between the realized and efficient unemployment rates: $u - u^*$. Both vacancies v and unemployment u have social costs associated with them and the efficient level of unemployment is the optimal allocation between vacancies and unemployment. The two variables are linked together through the Beveridge curve: $v = v(u)$, which gives the level of vacancies as a function of unemployment. More formally, the efficient level of unemployment u^* is the solution to a social planner's problem of maximizing social welfare $\mathcal{W}$ subject to the Beveridge curve:

$$\max_{u} \quad \mathcal{W} \quad \text{s.t.} \quad v = v(u). \tag{1}$$

The main contribution of Michaillat and Saez (2021) is their derivation of a sufficient statistics formula for the efficient unemployment rate. They show that the optimal allocation between vacancies and unemployment can be characterized by three sufficient statistics: recruiting costs, social cost of unemployment and elasticity of the Beveridge curve. Along with the knowledge of the current location of the Beveridge curve, these statistics are sufficient to solve for the efficient rate of unemployment. In the following we glance through the derivation of the sufficient statistics formula, borrowing heavily from the original paper.

As already mentioned, social welfare $\mathcal{W}$ is decreasing in both vacancies and unemployment:

$$\frac{\partial \mathcal{W}}{\partial v} < 0, \quad \frac{\partial \mathcal{W}}{\partial u} < 0.$$

The cost associated with additional vacancies are the costs of recruiting. In welfare sense, the relevant measure of this marginal cost is the lost welfare from additional employment n that those involved in the recruiting process could contribute instead. Let this measure be labeled as κ, which is normalized to be positive:

$$\kappa = -\frac{\partial \mathcal{W}/\partial v}{\partial \mathcal{W}/\partial n} > 0. \tag{2}$$

On the other side of the labour market, there are social costs related to unemployment. Unemployed people contribute less to social welfare than employed workers do, meaning that the social value of nonwork ζ, as defined below, is less than one:

$$\frac{\partial \mathcal{W}}{\partial u} < \frac{\partial \mathcal{W}}{\partial n} \implies \zeta \equiv \frac{\partial \mathcal{W}/\partial u}{\partial \mathcal{W}/\partial n} < 1.$$

The social cost of unemployment in the context of the welfare function is then given simply as

$$\frac{(\partial \mathcal{W}/\partial n) - (\partial \mathcal{W}/\partial u)}{\partial \mathcal{W}/\partial n} = 1 - \zeta > 0, \tag{3}$$

which measures the welfare loss from having a person unemployed rather than employed at the margin. This loss should reflect both the lost market production net of increased home production and recreation as well as other costs such as psychological pain incurred by being unemployed (Michaillat and Saez, 2021).

The third statistic needed to calculate the efficient trade-off between vacancies and unemployment is Beveridge elasticity which is the elasticity of the vacancy rate w.r.t. unemployment rate along the Beveridge curve (normalized to be positive):

$$\epsilon = -\frac{d\ln(v(u)))}{d\ln(u)} = -\frac{u}{v}\ v'(u). \tag{4}$$

In essence, this parameter determines the rate at which unemployment and vacancies can be traded off. Whereas the first two statistics are set according to available estimates in the literature, Beveridge elasticity is rather straightforwardly estimated using data on vacancy and unemployment rates.

Generally the above statistics might depend on unemployment and vacancy rates. To account for this endogeneity would make solving for the sufficient statistics formula more difficult. To simplify matters, Michaillat and Saez (2021) use the workaround developed by Kleven (2021) and assume that the statistics do not depend on unemployment and vacancy rates. We follow this simplifying assumption, although the sufficient statistics formula could be obtained also more generally with additional statistics that would inform us of the elasticities of the statistics themselves.

With the knowledge of the three statistics along with the above assumption, solution to the social planner's problem can be characterized explicitly. Consider again equation (1) and the associated FOC where we can now plug-in the welfare costs on unemployment and vacancies:

$$\frac{\partial \mathcal{W}}{\partial u} + \frac{\partial \mathcal{W}}{\partial v}\frac{\partial v}{\partial u} = 0 \Leftrightarrow -(1-\zeta) - \kappa v'(u) = 0. \tag{5}$$

Furthermore, we can combine this with (4) to arrive at

$$-(1-\zeta) - \kappa\epsilon(-\frac{v}{u}) = 0 \Leftrightarrow \frac{v}{u} = \frac{1-\zeta}{\epsilon\kappa} \Rightarrow \theta^* \equiv \frac{1-\zeta}{\epsilon\kappa}, \tag{6}$$

where θ^* is the efficient labour market tightness that follows from the FOC and is a function of the three statistics.

From the assumption that the statistics do not depend on unemployment and vacancy rates also follows that the Beveridge curve is isoelastic:

$$v(u) = a \cdot u^{-\epsilon}. \tag{7}$$

Parameter a determines the location of the curve and ϵ the steepness of it. Given knowledge of the position of the curve a and efficient labour market tightness θ^* we can calculate the efficient unemployment rate which is located on the curve at the point where tightness is efficient. On the curve, tightness is given by:

$$\frac{v(u)}{u} = a \cdot u^{-(1+\epsilon)}. \tag{8}$$

Now combining this with $\theta^* = \frac{1-\zeta}{\epsilon\kappa}$ yields a solution for the efficient unemployment rate u^*:

$$u^* = \left(\frac{a}{\theta^*}\right)^{1/(1+\epsilon)} = \left(a\frac{\epsilon\kappa}{1-\zeta}\right)^{1/(1+\epsilon)}. \tag{9}$$

Michaillat and Saez (2021) also assume that the economy is always on the Beveridge curve from which follows that position of the curve at time t, a_t, can be solved for using u_t and v_t: $a_t = v_t/u_t^{-\epsilon}$. Therefore, efficient unemployment rate at time t can be written as

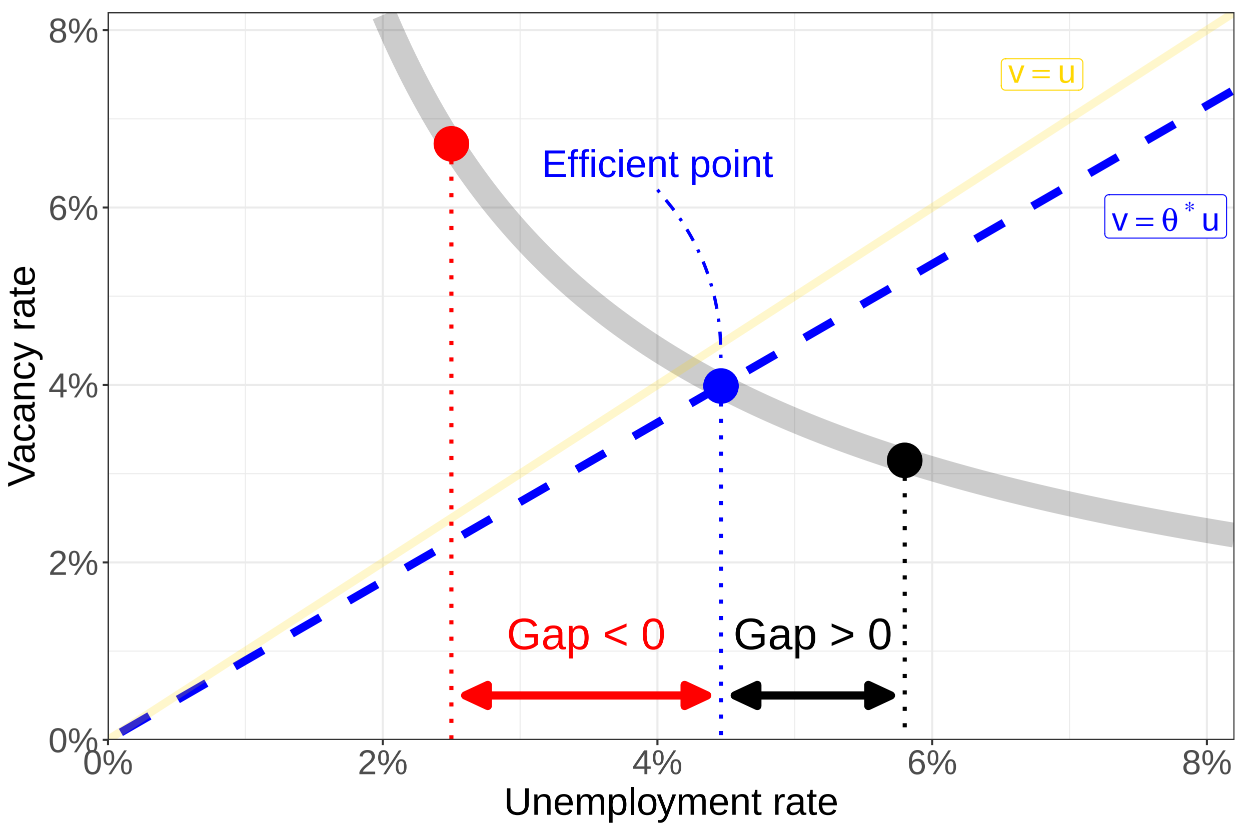
$$u_t^* = \left(\frac{v_t}{u_t^{-\epsilon}}\frac{\epsilon\kappa}{1-\zeta}\right)^{1/(1+\epsilon)}. \tag{10}$$

Beveridgean unemployment gap is given by the difference between realized unemployment rate and the efficient one: $u_t - u_t^*$.

2.2 Efficiency on the Beveridge curve

Figure 1 illustrates graphically how efficiency is determined on the Beveridge curve in the (u, v)-space. Given the position of the Beveridge curve (grey curve), the efficient point is simply found at the point where efficient labour market tightness θ^* is satisfied. As the blue dashed line gives all such points, the solution is found where this line and the Beveridge curve cross. This point is given by the blue dot and this point determines the level of efficient unemployment rate (as well as the associated efficient vacancy rate). Two other points on the curve are highlighted. At the point of the red dot labour markets are too tight and the unemployment gap is negative. Social welfare could be improved by trading off vacancies and their associated costs for increased unemployment. On the black dot labour markets are too slack and the unemployment gap is positive. Social welfare could be increased by lowering unemployment and increasing vacancies along the Beveridge curve.

Figure 1: Efficiency on the Beveridge curve.



Notes: Grey curve is a hypothetical Beveridge curve. Blue dashed line is given by the efficient labour market tightness $\theta^* = \frac{1-\zeta}{\epsilon\kappa}$. Blue dot at the point where these two curves cross is the efficient point. Red and black dots illustrate points on the Beveridge curve with inefficiently tight and slack labour markets respectively.

2.3 Parameters

Recruiting cost κ

In a brief review of hiring cost literature, Manning (2011) cites the difficulty of getting direct data and well-defined estimates on hiring costs. Firstly, there is a scarcity of employer surveys that document hiring or labour turnover costs. Also, for the purposes of the current analysis it is not always clear how well surveyed costs reflect the true costs that should be included in κ. Hiring costs can be thought to include both vacancy costs which are the costs of recruitment incurred before the new worker starts her job as well as output costs as the new worker is trained and she adapts to her job.

In their paper, Michaillat and Saez (2021) use the National Employer Survey from 1997 in combination with unemployment and vacancy rates for the US from the same year to infer the size of recruiting cost parameter. In the Survey, the mean response to the question of how large a share of labour costs are devoted to recruiting is 3.2%. Under the assumption that recruiters and other workers are paid the same wage, recruiting costs can be written on both sides of the equation as: $\kappa v = 3.2\% \cdot (1-u)$. Solving for κ and plugging in $v_{1997} = 3.3\%$ and $u_{1997} = 4.9\%$ yields an estimate: $\kappa = 3.2\% \cdot (1 - 4.9\%)/3.3\% = 0.92$. With $\kappa = 0.92$ it takes 0.92 recruiters to serve a vacancy.

There are few survey estimates on hiring costs from European economies. Blatter et al. (2012) use Swiss establishment-level survey data to show that the average cost of hiring skilled workers is 12.9 weeks of wages. The surveys were conducted in 2000 and in 2004 and they included both recruitment and adaptation costs. Muehlemann and Pfeifer (2016) find from a similar survey in Germany (in 2007) that average hiring costs are roughly 8 weeks of wage payments. As a percentage of wage bill these estimates correspond to 3.3% in Switzerland and 1.9% in Germany (Muehlemann and Pfeifer, 2016). Muehlemann and Strupler Leiser (2018) use a more recent vintage of the Swiss survey from 2009 that includes also disruption costs and estimate that average hiring costs are almost 16 weeks of wage payments.[3] By subtracting disruption costs (25.5% on average) the estimate would translate to under 12 weeks of wages which is approximately in line with the previous Swiss surveys covered in Blatter et al. (2012). Interestingly Muehlemann and Strupler Leiser (2018) also find evidence that costs of hiring depend on labour market tightness with higher v/u being associated with higher search costs.

As both the German and Swiss data provide also a breakdown of different costs associated with hiring, Faccini and Yashiv (2022) consider these data as the most detailed information available on hiring costs. They document how the output costs of hiring that result from time spent on interviews, training and disruption etc. are much larger than pecuniary costs of the vacancy and how the bulk of the costs are post-match costs rather than prematch. They report also that in a more recent German survey from 2012-2013 disruption costs amount to 57% of hiring costs (Faccini and Yashiv, 2022, Table 1) with total costs amounting to 3 months of wages of a newly hired worker. Considering the evidence in these surveys and taking a similar approach to Michaillat and Saez (2021) in translating these survey results into estimates of κ we get: $0.53-0.71$ for Switzerland and $0.58-1.35$ for Germany, depending on the costs involved.[4]

Bertheau et al. (2022) study labour turnover costs without relying on survey data. Instead they use administrative data from Denmark on unexpected worker deaths and firm outcomes to estimate that turnover costs amount to roughly one year of labour costs of an average employee. They match firms with an unexpected death of a worker to similar control firms based on firm characteristics and then compare differences in firm profits in the following years after the unexpected death of a worker. While the reported estimates of turnover costs

[3] Disruption costs arise from the time spent by other workers in introducing new hires to the production process. During this time these workers are disrupted from performing their regular productive tasks.

[4] Switzerland: hiring costs 3.3% of the wage bill in the sample of Blatter et al. (2012) (Muehlemann and Pfeifer, 2016), $v/u = 1.45$ in the sample of Muehlemann and Strupler Leiser (2018) (no discernible differences in v/u using aggregate data on vacancies (from the Swiss Federal Statistical Office) and unemployment (from OECD) during the different sample periods of Blatter et al. (2012) and Muehlemann and Strupler Leiser (2018)), $u = 4.1\%$ on average during the sample period for Blatter et al. (2012) (OECD Economic Outlook 112), $\kappa = 3.3\% \cdot (1-4.1\%)/(4.1\% \cdot 1.45) = 0.53$, including disruption costs (25.5%) $\kappa = 0.53/(1-0.255) = 0.71$; Germany: hiring costs 1.9% of the wage bill (Muehlemann and Pfeifer, 2016), $u_{2007} = 8.7\%$ and $v_{2007} = 3.0\%$ (Eurostat), $\kappa = 1.9\% \cdot (1-8.7\%)/3.0\% = 0.58$, including disruption costs (57%) $\kappa = 0.58/(1-0.57) = 1.35$.

in Bertheau et al. (2022) are larger than most of the survey-based estimates, they are not perhaps fully comparable given that the estimates do not separate foregone output from hiring costs and the worker exits studied are not necessarily representative of an average worker separation. In contrast, survey-based estimates might miss some of the costs.

Lacking definitive evidence on the size of κ, we use as a baseline the same value used by Michaillat and Saez (2021): $\kappa = 0.92$. Subsection 4.4 provides robustness analysis using different values for all parameters, including recruiting costs. Also, studying the possible implications of endogenizing this parameter with respect to labour market tightness, given the evidence in Muehlemann and Strupler Leiser (2018), is left for future work.

Social value of nonwork ζ / Social cost of unemployment $1 - \zeta$

Michaillat and Saez (2021) build their measure of the social value of nonwork on revealed-preference estimates available in the literature. More specifically, they translate estimates from two empirical papers: Borgschulte and Martorell (2018) and Mas and Pallais (2019); into estimates of the social value of nonwork.[5] Both of these papers use US data but since the parameter of interest should in theory reflect the nonpecuniary value of nonwork, the external validity of these estimates with respect to Europe should in principle not be influenced by e.g. different levels of unemployment benefits between the US and European economies.

Borgschulte and Martorell (2018) study the decisions of US military service members between reenlisting and exiting the military. They utilize variation in local labour market conditions and in occupation-specific reenlistment bonuses to estimate the willingness of service members to avoid having to reenter the civilian labour market at a time of higher unemployment. They find that between 13% and 35% of estimated earnings losses due to higher unemployment during transition to civilian employment are offset by leisure, home production, public benefits or other mitigating factors. To arrive at an estimate of the social value of nonwork, Michaillat and Saez (2021) make various adjustments to this estimate in

[5]The fact that employed and unemployed workers value consumption differently is ignored in (Michaillat and Saez, 2021) allowing to measure the contribution of workers to welfare directly from their productivity.

order to account for factors like UI benefits that the individual service member faces but should not be reflected in ζ. They deduce that the estimates imply a social value of nonwork between 0.03 and 0.25.[6]

Mas and Pallais (2019) on the other hand use a field experiment to estimate the marginal value of nonwork time. In the experiment, job applicants were offered randomized wage-hour bundles. Using observed choices of unemployed jobseekers it is possible to estimate the marginal value of time at different levels of work hours — tracing out a labour supply relationship. By combining estimates of the marginal value of time with predicted market wages of workers, they estimate that nonwork time is worth 58% relative to pretax earnings. Again, Michaillat and Saez (2021) adjust this original estimate in order for it to reflect the desired parameter ζ. They conclude that the estimates of Mas and Pallais (2019) imply a ζ between 0.41 and 0.49.

With the estimates of $0.03 - 0.25$ and $0.41 - 0.49$ from the two papers at hand, Michaillat and Saez (2021) decide to use a midrange value of $\zeta = 0.26$ from the plausible range of $0.03 - 0.49$. The authors point out that as these estimates are based on revealed preference choices of individual agents, they might miss externalities, like increased crime due to higher unemployment, imposed by nonwork. As a baseline, we follow the original paper and set $\zeta = 0.26$.

Beveridge elasticity ϵ

Michaillat and Saez (2021) estimate the Beveridge elasticity by regressing log vacancy rate on log unemployment rate. However, since in their sample (1951Q1-2019Q4) there are apparent

[6]Estimates are first transformed to be expressed in terms of marginal product of labour which is taken to be 3% to 25% higher than the wage. Then a 7.7% employer-side payroll tax is accounted for. Also the value of public benefits must be subtracted from the original estimates. Chodorow-Reich and Karabarbounis (2016) find that UI benefits amount to 21.5% and other public benefits to 2% of the marginal product of labour. However, since UI takeup rate is only 65%, UI benefits and consumption are taxed, UI benefits expire and filing for benefits causes disutility; the average value of UI benefits is taken to be 5% of the marginal product of labour. Therefore, in total 7% of the marginal product of labour is subtracted due to public benefits. All these transformations mean that the estimates of Borgschulte and Martorell (2018) imply a social value of nonwork between $\frac{\mathbf{0.13}}{\mathbf{1.25}\cdot 1.077} - 0.07 = 0.03$ and $\frac{\mathbf{0.35}}{\mathbf{1.03}\cdot 1.077} - 0.07 = 0.25$.

shifts in the Beveridge curve across periods, they utilize the algorithm of Bai and Perron (1998, 2003) to account for multiple structural breaks. The algorithm splits the sample into subsamples by finding breakpoints in the data and then in each of the subsamples the model is estimated separately. Elasticity estimates of Michaillat and Saez (2021) are in the range of $0.84 - 1.02$ (averaging 0.91 over the whole sample). Thus it seems that the Beveridge elasticity is rather stable in the US data while there are changes in the location of the Beveridge curve in the (u, v)-space over time. Not accounting for these shifts would bias the estimates of ϵ.[7]

Given that we are working with rather short (internally consistent) time series in the case of European economies (see section 3 for details on the data), splitting the data into multiple subsamples is not feasible. Instead, we estimate Beveridge elasticity using the full sample that we have at our disposal and then assess whether potential shifts in the Beveridge curve are a cause for concern in our case. Empirical estimates for European economies are reported in subsection 4.1.

2.4 $u^* = \sqrt{uv}$

In a more recent paper Michaillat and Saez (2022) argue for an even more simple measure of the efficient unemployment rate: $u^* = \sqrt{uv}$. This is a special case of the more general formula for u^* with certain parameter values. Namely, they assume that both creating vacancies and being unemployed have similar social costs and thus deviate from Michaillat and Saez (2021). From this follows that the nonproductive use of labour is well measured by the sum of vacancies and unemployment. Also, as the social costs are equal, $\kappa = 1 - \zeta$, these two parameters cancel out from (10) and the equation can be written as

$$u_t^* = \left(\epsilon \frac{v_t}{u_t^{-\epsilon}}\right)^{1/(1+\epsilon)}, \tag{11}$$

[7]See Ahn and Crane (2020) for an approach that aims to dynamically account for both movements along a stable Beveridge curve as well as time variation in factors that shift the curve.

instead.[8]

To arrive at $u_t^* = \sqrt{uv}$, Beveridge curve is assumed to be a rectangular hyperbola: $v(u) = \alpha \cdot u^{-1}$ with $\epsilon = 1$. This again deviates from Michaillat and Saez (2021) but in the US context estimates of ϵ are quite close to unity in the first place and the Beveridge curve may be well represented by a hyperbolic function. It is an empirical question whether the relationship between vacancies and unemployment is well approximated by a hyperbola in other economies and thus whether $\epsilon = 1$ is a reasonable assumption. In subsection 4.5 we contrast measures of u^* to both $\sqrt{uv}$ and the one given by (11) in the case of Finland.

3 Data

We collect quarterly data on unemployment and vacancy rates from Eurostat. Unemployment rate is from the Labour Force Survey (LFS) and calculated by dividing the number of unemployed persons by the number of active persons in the 15-74 year old labour force. Vacancy rate is measured by dividing the number of survey-based (non-farm) vacancies in Job vacancy statistics by the same active population as in the case of unemployment.[9] All series are seasonally adjusted by Eurostat.

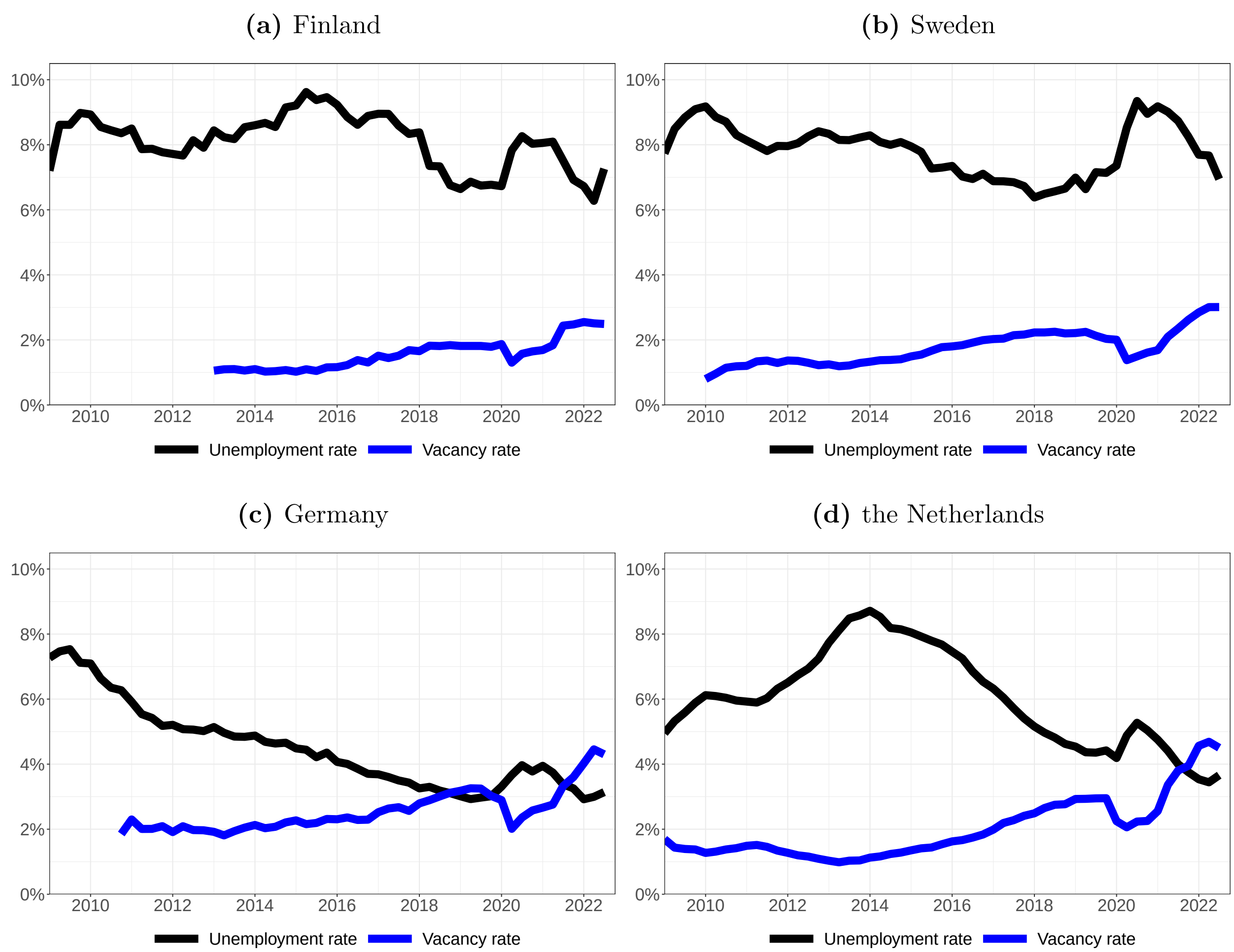
The main sample consists of 2009Q1-2022Q3 for which internally consistent LFS time series are available from Eurostat. For vacancies the availability of data without breaks for this period in Job vacancy statistics is mixed. For the Netherlands, seasonally-adjusted series are available for the whole period, while for other countries the samples are shorter: for Finland 2013Q1-, for Sweden 2010Q1- and for Germany 2010Q4-. Figure 2 plots the data on unemployment and vacancy rates.

For Finland, we also extend this sample with archived OECD data that are accessible through FRED in order to provide historical perspective to the Finnish labour market already

[8] In Michaillat and Saez (2021) $\kappa > 1 - \zeta$ with $\frac{0.92}{1-0.26} \approx 1.24$, meaning that setting $\kappa = 1 - \zeta$ decreases u^* relative to the original paper.

[9] The term non-farm here refers to NACE Rev. 2 B-S.

Figure 2: Unemployment and vacancy rates, 2009Q1-2022Q3.



Notes: All data are from and seasonally-adjusted by Eurostat. Unemployment rate is calculated by dividing the number of unemployed persons by the number of active persons in the Labour Force Survey (LFS). Vacancy rate is calculated by dividing the number of non-farm (NACE Rev. 2 B-S) job vacancies in Job vacancy statistics by the number of active persons in LFS.

from 1964Q1.[10] In order to do this we splice the more recent series with these data such that the levels would reflect the more recent survey-based measures at the point of the data break in 2013. However, in estimating the Beveridge elasticity for Finland we limit ourselves to the more recent, survey-based and internally consistent series for 2013Q1-2022Q3 and consider this the main sample for the analysis. Estimates based on this sample are then applied to the historical series.

4 Results

This section reports results of the empirical analysis. We study four European economies in Finland, Sweden, Germany and the Netherlands using data for 2009Q1-2022Q3. For Finland, we provide results also for a longer time period that starts from the 1960s.

4.1 Beveridge elasticity estimates

Beveridge elasticity is estimated by fitting a log-linear Beveridge curve:

$$\ln(v_t) = \alpha - \epsilon \ln(u_t) + z_t \tag{12}$$

to data. Table 1 presents estimates of the Beveridge elasticity for the countries studied. For Finland and Sweden we find elasticities of 1.94 and 1.82 respectively. These estimates are closer to 2 than to unity and the Beveridge curve is thus empirically more steep than in the United States. In contrast, the estimate for Germany of 0.92 is close to those obtained by Michaillat and Saez (2021) for the US. The Dutch estimate of 1.51 is somewhere between these two.

[10] These series are FINURTOTQDSMEI: Unemployment Level: Survey-Based (All Persons) in Finland (DISCONTINUED); FINLFTOTQDSMEI: Civilian Labor Force: All Persons in Finland (DISCONTINUED); LMJVTTUVFIQ647S: Labour - Other Labour Market Measures: Job Vacancies: Total: Unfilled Vacancies (Stock) for Finland. All these data are from OECD Main Economic Indicators (database) and seasonally-adjusted. Original source for these data are national authorities: Statistics Finland, Ministry of Economic Affairs and Employment.

Table 1: Beveridge elasticity estimates.

	Dependent variable: $\ln(v)$			
	Finland	Sweden	Germany	Netherlands
	(1)	(2)	(3)	(4)
$-\ln(u)$	1.94***	1.82***	0.920***	1.51***
	(0.342)	(0.567)	(0.156)	(0.155)
Constant	-9.09***	-8.74***	-6.65***	-8.28***
	(0.859)	(1.50)	(0.491)	(0.452)
Observations	39	51	48	55
Adjusted R^2	0.629	0.377	0.743	0.801
Sample period	2013Q1-2022Q3	2010Q1-2022Q3	2010Q4-2022Q3	2009Q1-2022Q3

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$

Notes: Newey-West standard errors with 4 lags in parentheses.

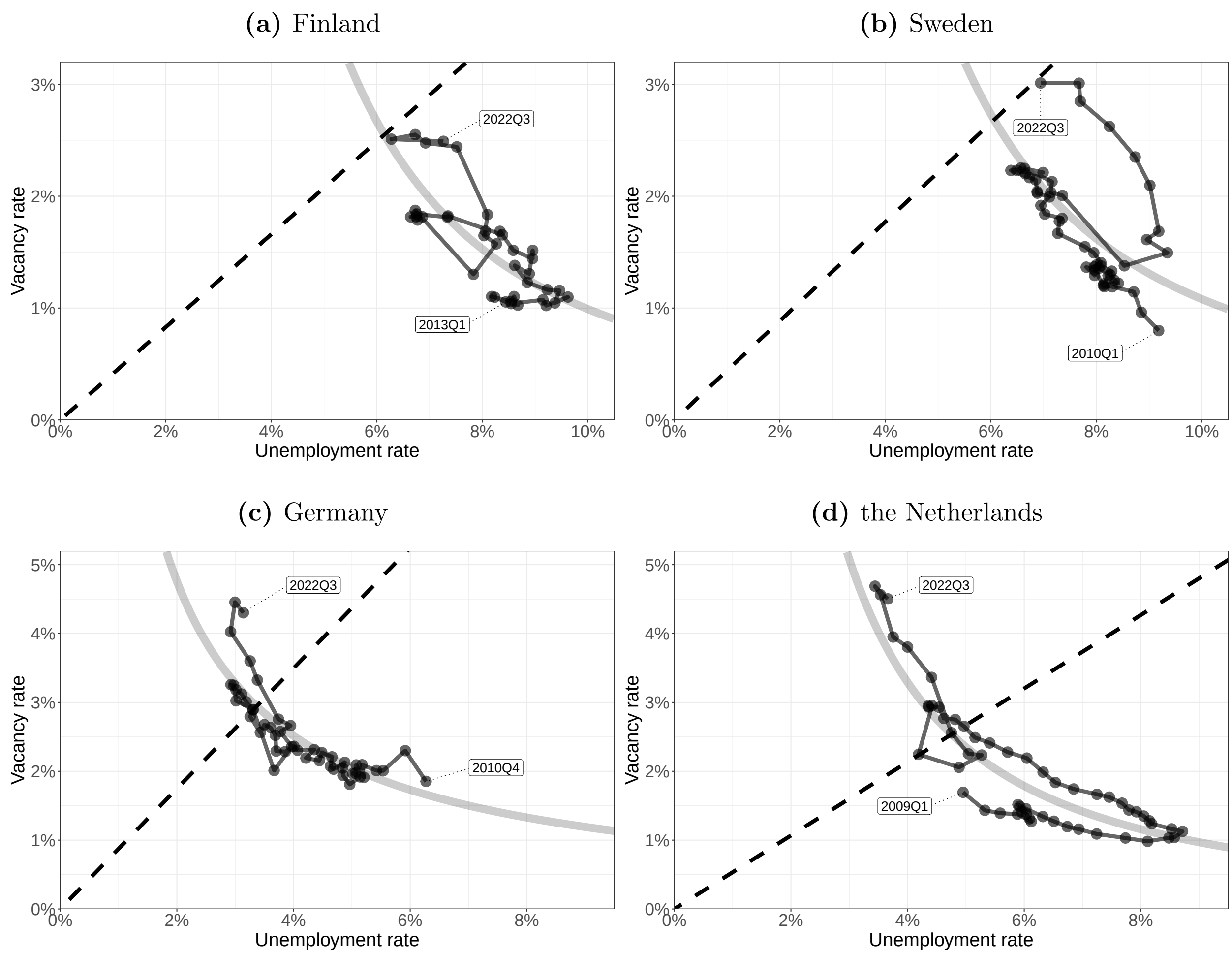
As alluded to in subsection 2.3, shifts in the Beveridge curve may be detrimental to estimation of the elasticity. In this respect the samples here seem to work reasonably well with no large shifts until the most recent period of observable data (see Figure 3). The apparent outward shift in Sweden might to some degree affect estimates of ϵ in this small a sample. It is also worthwhile to note that compared to Germany and the Netherlands, there is relative little variation in the unemployment rate in both the Finnish and Swedish samples.

4.2 Empirical Beveridge curve

Figure 3 plots the observed vacancy and unemployment rates along with the average Beveridge curve in the Finnish, Swedish, German and Dutch samples. Steepnesses of the curves are determined by estimated elasticities for each country and the average position by their observable data. Dashed lines give the efficient labour market tightness for each country. With the estimated elasticities from Table 1 these efficient levels of v/u are: 0.41 for Finland, 0.44 for Sweden, 0.87 for Germany and 0.53 for the Netherlands.

In each country, observed vacancy rates have been historically high in the latter part of

Figure 3: Empirical Beveridge curves.



Notes: Position of the grey curve is given by the average position of the Beveridge curve in this given sample. Dashed line is the efficient labour market tightness.

the sample. Whereas both Germany and the Netherlands have seen v/u higher than unity and therefore above the efficient level in recent times, in Finland and Sweden the highest observed levels of labour market tightness just about correspond to the efficient level with rest of the sample being below the efficient level.

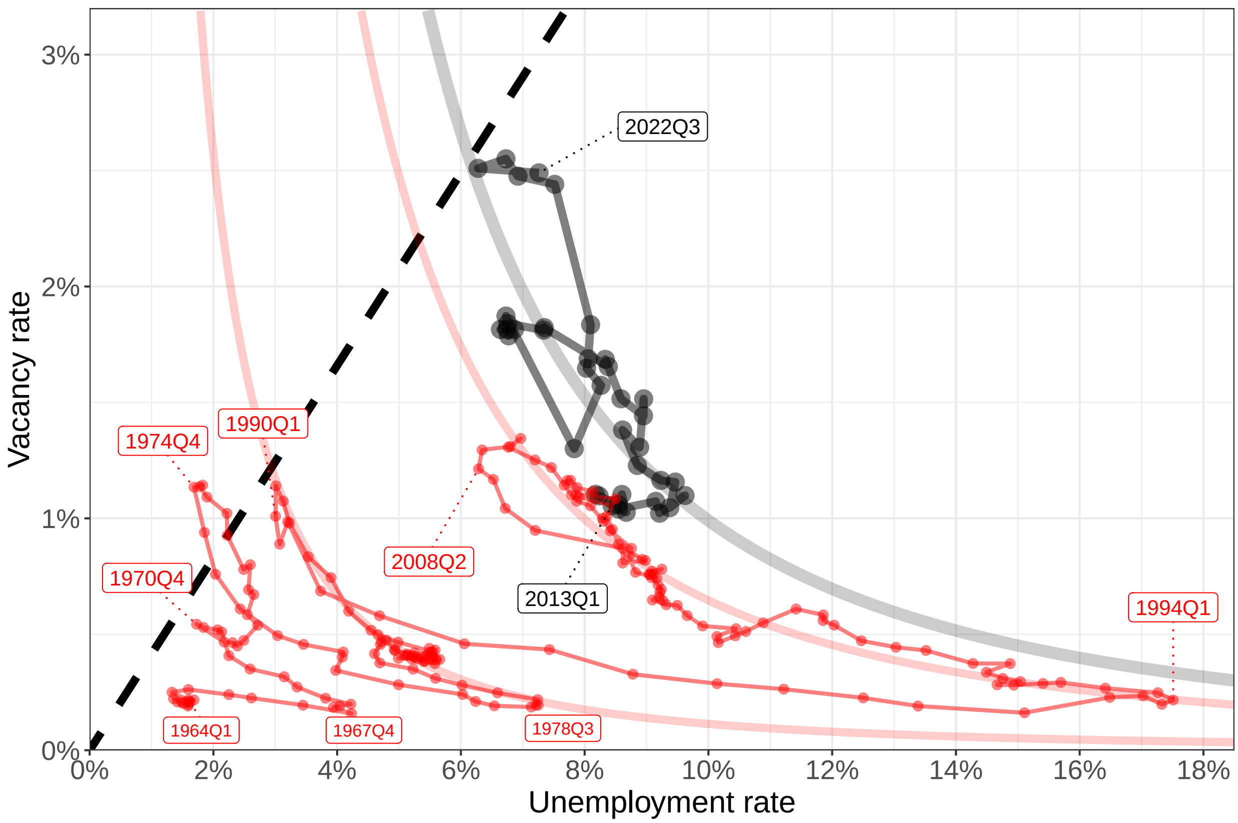
For Finland, we present the Beveridge diagram also for a longer sample in Figure 4. In this figure, the sample has been continued backwards to 1964Q1 as discussed in Section 3. The graph naturally contains the more recent sample (2013Q1-2022Q3) as before (grey curve and black dots). In addition, observations belonging to the extended sample are plotted as red dots.

We have also included two other Beveridge curves in Figure 4. These correspond to 1994Q1-2012Q4 and 1978Q1-1990Q4 as the relation between vacancies and unemployment seems quite stable during these two periods with no large shifts in the position of the curve. The Beveridge curve for 1994Q1-2012Q4 seems to be a remarkably good fit for the data all the way back to 1994Q1 which saw the unemployment rate peak at almost 18% during the Finnish depression. Considering that the unemployment rate varies roughly between 6.5% and 9% in the sample that the Beveridge elasticity is estimated in, the fact that the curve is a good fit also for observations out-of-sample and well outside the range of observations in the estimation sample is noteworthy.

Evidently, there is a large shift in the Finnish Beveridge curve during the 1990s depression which is also visible in Figure 4. In 1990 unemployment was roughly 3% and Finnish labour market was almost efficiently tight but during the early 90s depression the empirical Beveridge clearly shifts outwards. On the Beveridge curve that seems to fit the 1994Q1-2012Q sample, vacancy rates that were consistent with an unemployment rate of 3% in 1990 would translate to unemployment rates of approximately between 7% and 8%.

In the historical perspective, the shift of the Beveridge curve during the early 90s is not the only one but it is perhaps the largest. Between the late 1970s and early 1990s the relationship between vacancy and unemployment rates seems also quite stable and observations fall again

Figure 4: Beveridge curve, Finland 1964Q1-2022Q3.



Notes: Slope of the Beveridge curve is estimated with 2013Q1-2022Q3 as the sample (black dots) and position of the grey curve is given by the average position of the Beveridge curve in this same sample. Red dots are the extended sample (1964Q1-2012Q4) which has been spliced using register-based vacancies prior to 2013 and older vintages of survey-based unemployment and labour force prior to 2009. Dashed line is given by the efficient labour market tightness. Labeled points correspond either to breakpoints in the data or quarters with local minima/maxima in the unemployment rate.

on a Beveridge curve with the elasticity estimated in the 2013Q1-2022Q3 sample and the average position given by observations between 1978Q1 and 1990Q4. For further evidence of the same Beveridge elasticity being a reasonable assumption, Table 2 presents elasticity estimates using the aforementioned periods 1994Q1-2012Q4 and 1978Q1-1990Q4 from the extended sample. The estimates for these periods are 1.78 and 1.81 respectively, compared to 1.94 for the 2013Q1-2022Q3 sample. Similarly to the US case, a constant Beveridge elasticity seems to be a reasonable assumption even though the position of the curve may shift over time.

Table 2: Beveridge elasticity estimates for Finland.

	Dependent variable: $\ln(v)$		
	(1)	(2)	(3)
$-\ln(u)$	1.94***	1.78***	1.81***
	(0.342)	(0.086)	(0.093)
Constant	-9.09***	-9.15***	-10.9***
	(0.859)	(0.198)	(0.290)
Observations	39	76	52
Adjusted R^2	0.629	0.934	0.940
Sample period	2013Q1-2022Q3	1994Q1-2012Q4	1978Q1-1990Q4

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$

Notes: Newey-West standard errors with 4 lags in parentheses. Columns (2) and (3) use register-based vacancies for vacancy rate and older vintage of survey-based measures for unemployment and labour force (prior to 2009Q1).

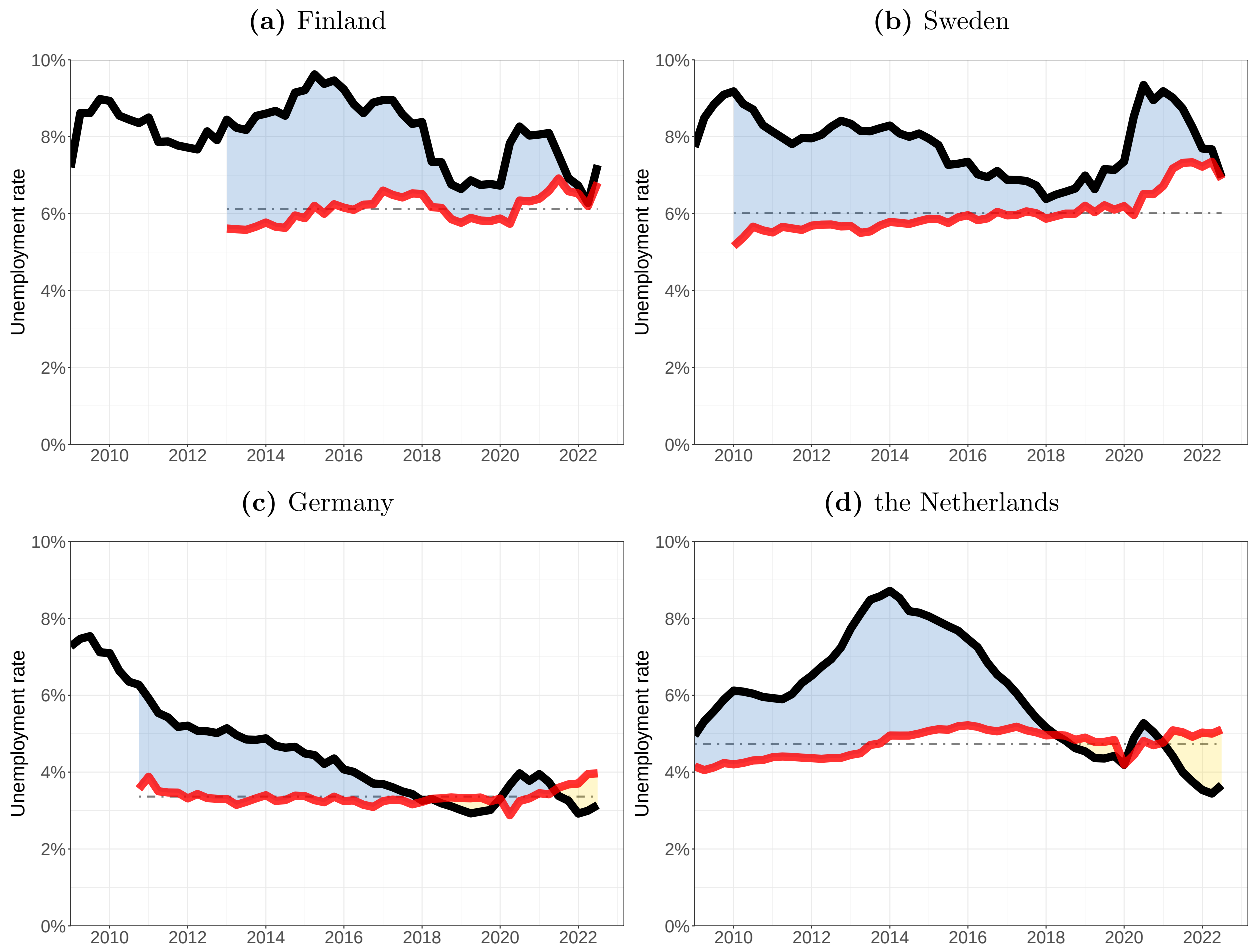
4.3 Unemployment gap

In Figure 5 we present the Beveridgean unemployment gaps for Finland, Sweden, Germany and the Netherlands. In the plot, black line is the seasonally adjusted quarterly unemployment rate and red line the efficient unemployment rate. The shaded area between these lines is blue whenever the labour market is inefficiently slack and yellow when it is inefficiently tight. The average efficient unemployment rate in the sample is given by the dot-dashed line. For the red line the only source of variation is the location of the Beveridge curve. By assumption all observations are on the Beveridge curve and thus the curve shifts every period according to observed vacancy and unemployment rates.

For Finland and Sweden we find quite similar results: on average the efficient rate of unemployment is around 6% and the unemployment gap is positive (labour markets are slack) with the unemployment rate fluctuating between roughly 6.5% and 9.5%. For Germany, the efficient unemployment rate is quite low: around 3.5% on average between 2010Q4-2022Q3. For the Netherlands the efficient unemployment rate is roughly 5%.

For Finland we again present a longer time series in Figure 6. Even with all the caveats

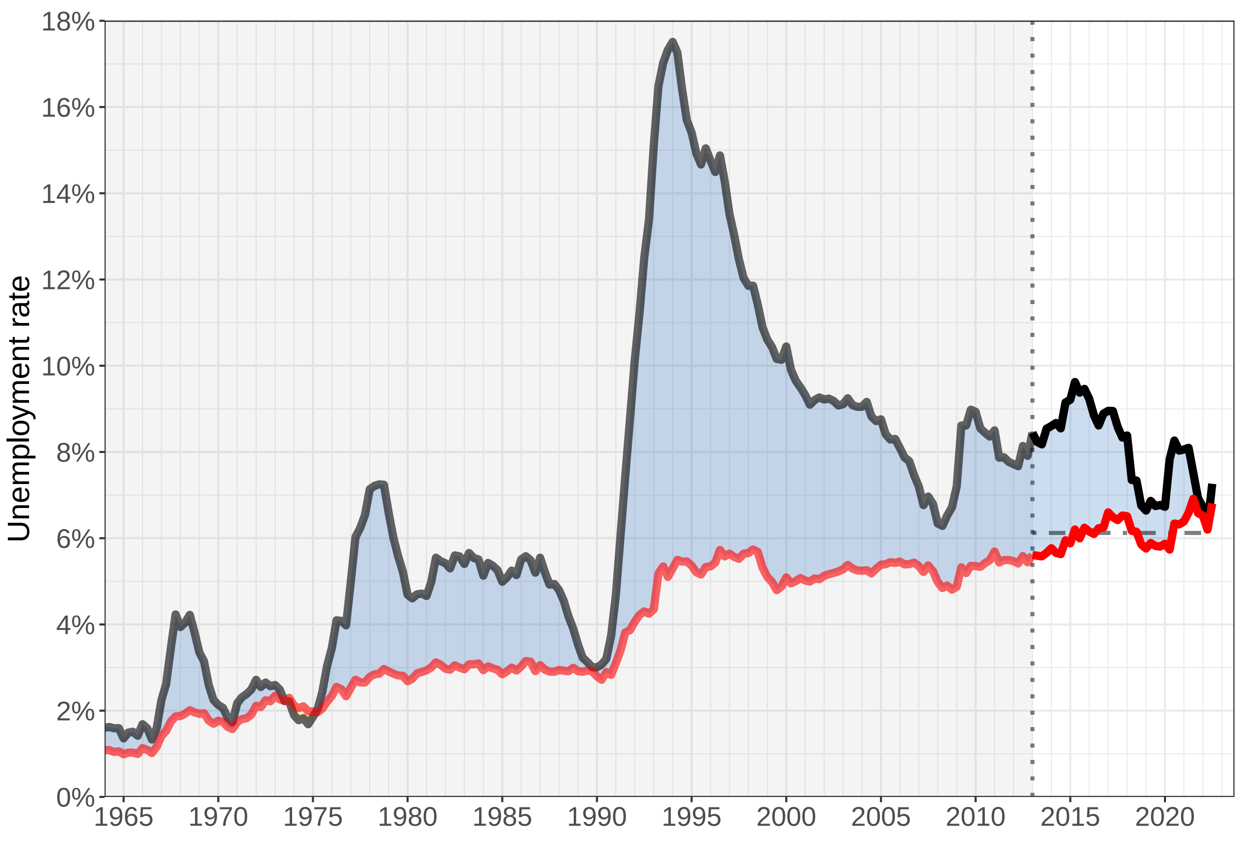
Figure 5: Beveridgean unemployment gaps, 2009Q1-2022Q3.



Notes: Black line is the seasonally adjusted quarterly unemployment rate. Red line is the efficient unemployment rate. The shaded area between these lines is blue whenever the labour market is inefficiently slack and yellow when it is inefficiently tight. The average efficient unemployment rate is given by the dot-dashed line.

about using older vintages of data in order to build the extended sample, it seems clear that Finnish labour market has been chronically slack historically. Only in the mid 1970s are we able to find a period in which the unemployment rate was below the efficient level and labour markets too tight. A large outward shift of the Beveridge curve during the early 90s depression is clearly observable in this graph as the efficient rate of unemployment jumps from around 3% to more than 5% over a period of few years. This is due to the Beveridge curve shifting outwards as can be observed in Figure 4. Periods prior to (1978Q1-1990Q4) and after (1994Q1-2012Q4) this shift see a relatively stable relationship between vacancies and unemployment and thus also a relatively stable u^*.

Figure 6: Beveridgean unemployment gap, Finland 1964Q1-2022Q3.



Notes: Black line is the seasonally adjusted unemployment rate. Red line is the efficient unemployment rate. The shaded area between these lines is blue whenever the labour market is inefficiently slack and yellow when it is inefficiently tight. Dotted vertical line represents the data break in 2013Q1. On the RHS of this line the average efficient unemployment rate between 2013Q1-2022Q3 is given by the dot-dashed horizontal line.

4.4 Robustness

To assess the robustness of our empirical results we vary each of the sufficient statistics one-by-one. Figure 7 plots the effects of differing parameter values both in the (u, v)-space and for the time series of the efficient unemployment rate separately. Here, the robustness analysis focuses solely on the Finnish case. Naturally, similar logic with respect to parameter values applies to other countries as well.

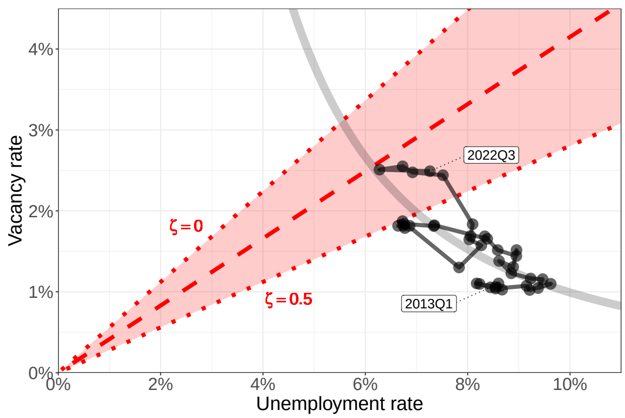
Firstly, we let the social value of nonwork ζ have values between 0 and 0.5 with $\zeta = 0.26$ still being the baseline. As social value of nonwork is independent from the Beveridge curve, the only channel for ζ to affect u^* is through its effect on the efficient labour market tightness θ^* and thus movements along the Beveridge curve. Since higher values of social value of nonwork imply that unemployment is socially less costly, θ^* is decreasing in ζ and higher (lower) values of ζ imply a higher (lower) u^*. The way in which different values of ζ affect efficient labour market tightness and thus unemployment on the Beveridge curve is illustrated in Figure 7a and a time series of efficient unemployment rates in Figure 7b. On average, $\zeta = 0$ yields an efficient rate of unemployment of roughly 5.5% and $\zeta = 0.5$ of close to 7%.

Given that the evidence on κ is somewhat vague given the literature on hiring costs which was briefly reviewed in subsection 2.3, there is perhaps no obvious way to construct a range of plausible values for this parameter. Here we let κ have values between $0.5 - 1.5$ with 0.92 naturally being the baseline. Higher recruiting costs κ imply that vacancies are socially more costly. Thus θ^* is decreasing in κ and higher (lower) values of κ imply a higher (lower) u^*. The effect on Finnish estimates can be seen in Figures 7c and 7d. On average, $\kappa = 0.5$ yields an efficient rate of unemployment of roughly 5% and $\kappa = 1.5$ of around 7.2%.

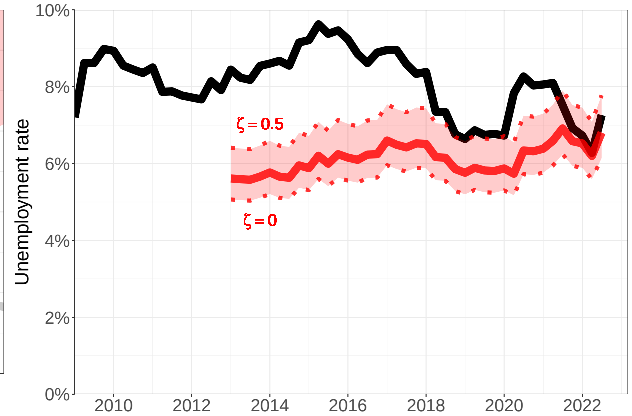
Lastly, for the Beveridge elasticity setting the range for robustness analysis follows naturally from empirical estimates and we set the range to 95% confidence interval for the elasticity based on estimates in Table 1: $1.25 - 2.63$. Here there are two channels for ϵ to affect estimates of u^*. Firstly, ϵ affects θ^* with higher values of ϵ meaning that the trade-off

Figure 7: Robustness analysis, Finland.

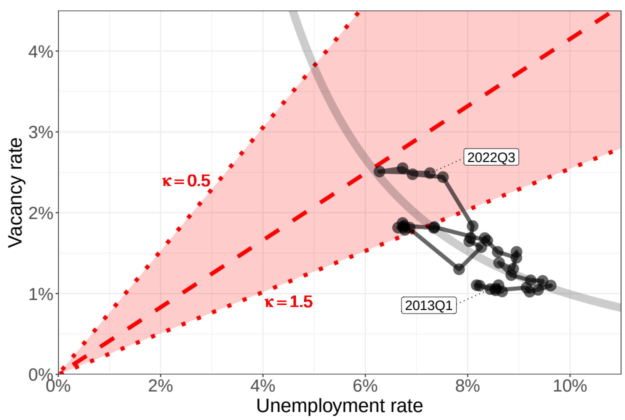
(a) Social value of nonwork: $0 \leq \zeta \leq 0.5$



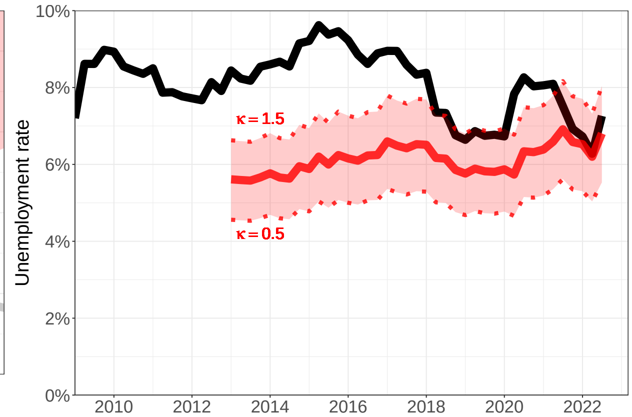
(b) Social value of nonwork: $0 \leq \zeta \leq 0.5$



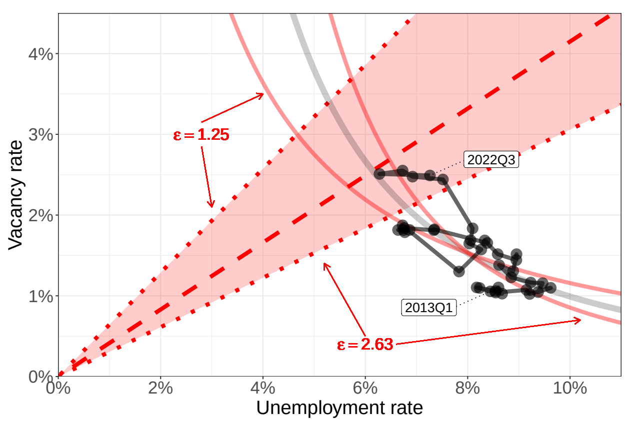
(c) Recruiting cost: $0.5 \leq \kappa \leq 1.5$



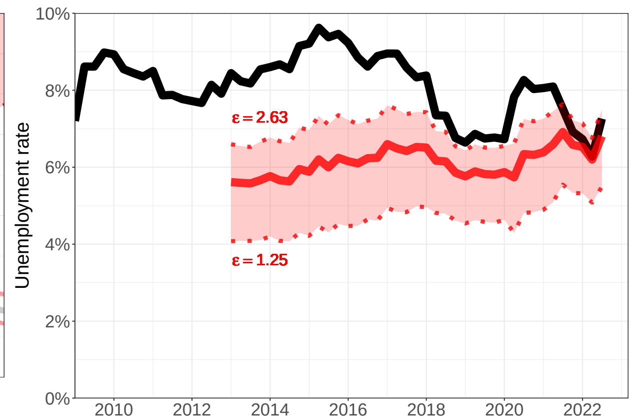
(d) Recruiting cost: $0.5 \leq \kappa \leq 1.5$



(e) Beveridge elasticity: $1.25 \leq \epsilon \leq 2.63$



(f) Beveridge elasticity: $1.25 \leq \epsilon \leq 2.63$



Notes: Baseline parameter values: $\zeta = 0.26$, $\kappa = 0.92$ and $\epsilon = 1.94$.

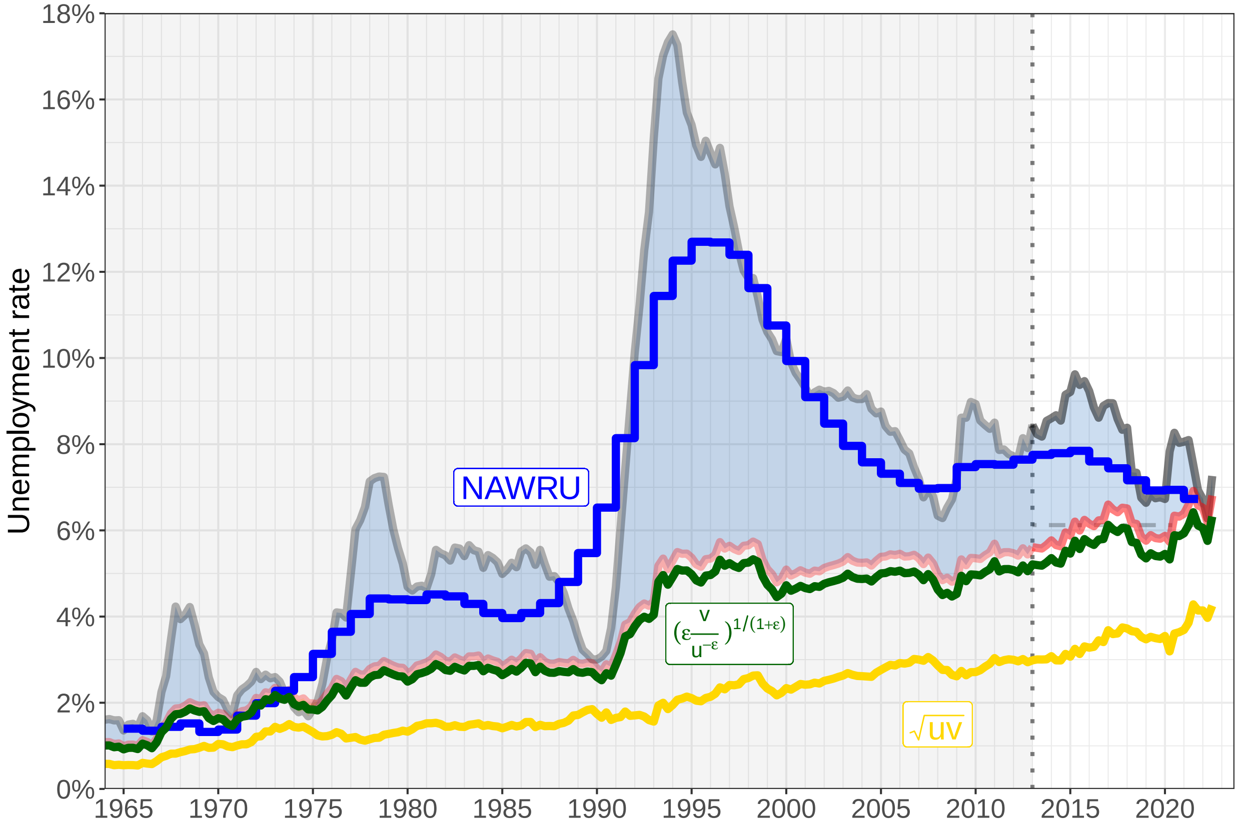
between vacancies and unemployment is more costly. Thus efficient labour market tightness θ^* is decreasing in ϵ. Secondly, also the Beveridge curve is affected as higher values of ϵ are associated with a steeper Beveridge curve. These both channels are illustrated in Figure 7e. Figure 7f plots the resulting estimates of u^* over time. On average, $\epsilon = 1.25$ yields an efficient rate of unemployment of roughly 4.6% while $\epsilon = 2.63$ results in estimates of around 7%.

4.5 Other measures of u^*

Figure 8 compares the efficient unemployment rate that the analysis above yielded against alternative measures of u^* in the extended sample for Finland. Underlying this figure is Figure 6. Over this background we plot: NAWRU as estimated by the European Commission (blue line) along with $u^* = \sqrt{uv}$ (yellow line) and its sibling given by (11) with the Beveridge elasticity given by its empirical estimate (green line). For NAWRU, data are available only at yearly frequency.

The non-accelerating wage rate of unemployment, NAWRU, is the result of a statisical decomposition where the NAWRU component is taken to represent a labour market equilibrium. The unobserved components approach used by European Commission is described in Hristov et al. (2017). In Figure 8, estimates of NAWRU seem to follow realized unemployment rate much more closely than the efficient unemployment rate that results from the sufficient statistics approach. A rise in unemployment during the late 1970s is accompanied by a rise in NAWRU whereas the Beveridgean approach interprets the rise mostly as a widening of the gap between the efficient level and realized unemployment. Curiously, NAWRU starts to rise already in advance of the 90s Finnish depression even as unemployment is declining at the same time in late 1980s. During high unemployment of early 1990s, estimates of NAWRU then rise above 12% before declining as realized unemployment rate falls. In the Beveridgean framework, the decline from record high unemployment in 1994 to levels seen in 2000s can be interpreted as a move along a rather stable Beveridge curve. If the location

Figure 8: Efficient rate of unemployment compared to alternative measures.



Notes: As in Figure 6 the black line is the seasonally adjusted unemployment rate and the red line is the efficient unemployment rate. Blue line: NAWRU is from European Commission and available at yearly frequency whereas other series are quarterly. Yellow line: $u^* = \sqrt{uv}$ as in Michaillat and Saez (2022). Green line: same assumptions as for the yellow line, except that $\epsilon = 1.94$ as estimated with 2013Q1-2022Q3 as the sample.

of the curve does not change, neither does the efficient level meaning that it is also rather stable over this period. On average, estimates of NAWRU are higher than efficient unemployment rates implied by the Beveridgean approach. Policymaker that takes NAWRU as her guide would therefore target higher levels of unemployment than a policymaker following the Beveridgean approach.

$u^* = \sqrt{uv}$ which is build as a more simple measure of u^* as explained in subsection 2.4 results in the lowest estimates of u^* in Figure 8. However, it is build under the assumption that $\epsilon = 1$ which is not the case for Finland given our estimates. In comparison, using the formula in (11) yields estimates of u^* that are close to the baseline. The only difference between $u^* = \sqrt{uv}$ (yellow line) and (11) (green line) lies in ϵ, whereas the only difference between (11) and baseline (red line) estimates of u^* stems from differences in κ and ζ parameters.

5 Conclusion

We apply the sufficient statistics approach of Michaillat and Saez (2021) in a European context to estimate Beveridgean unemployment gaps for Finland and three of its peers. Finnish labour market appears as persistently slack with large inefficiences during economic downturns. Over the last 10 years, the efficient unemployment rate in Finland has been around 6.1% on average. As unemployment rate has averaged 8.1% over the same period, Beveridgean unemployment gap has been roughly 2% on average.

Beveridgean unemployment gap gives a different picture of u^* in Finland than e.g. European Commission's NAWRU estimates. As an example, estimates of NAWRU are more than double the efficient level implied by the Beveridgean approach in the mid 1990s. Even though NAWRU is not explicitly defined in terms of optimal level of unemployment, it is occasionally considered as a guide for policy given that it gives an estimate of the structural rate of unemployment. It also has a role in policymaking through its link to estimates of potential output.

When viewed as a proxy measure of the business cycle, the Beveridgean unemployment gap bears resemblance to the plucking model of business cycles. According to this view business cycles are better characterized as occasional drops below potential output rather than symmetrical fluctuations around it (Dupraz et al., 2022). In our results, the efficient unemployment rate appears comparatively stable while the unemployment gap sees large fluctuations. Even though the Beveridgean unemployment gap doesn't necessarily have the same interpretation as the output gap, this property is potentially meaningful for the design of macroeconomic policy.

References

Ahn, Hie Joo, and Leland D. Crane. 2020. “Dynamic Beveridge curve accounting.” Finance and Economics Discussion Series 2020-027, Board of Governors of the Federal Reserve System.

Bai, Jushan, and Pierre Perron. 1998. “Estimating and testing linear models with multiple structural changes.” *Econometrica* 47–78.

Bai, Jushan, and Pierre Perron. 2003. “Computation and analysis of multiple structural change models.” *Journal of Applied Econometrics* 18 (1): 1–22.

Bertheau, Antoine, Pierre Cahuc, Simon Jäger, and Rune Vejlin. 2022. “Turnover costs: evidence from unexpected worker separations.” Job Market Paper.

Blatter, Marc, Samuel Muehlemann, and Samuel Schenker. 2012. “The costs of hiring skilled workers.” *European Economic Review* 56 (1): 20–35.

Borgschulte, Mark, and Paco Martorell. 2018. “Paying to avoid recession: Using reenlistment to estimate the cost of unemployment.” *American Economic Journal: Applied Economics* 10 (3): 101–27.

Chodorow-Reich, Gabriel, and Loukas Karabarbounis. 2016. “The cyclicality of the opportunity cost of employment.” *Journal of Political Economy* 124 (6): 1563–1618.

Dupraz, Stéphane, Emi Nakamura, and Jón Steinsson. 2022. “A plucking model of business cycles.” Working Paper, August 6, 2022.

Elsby, Michael W.L., Ryan Michaels, and David Ratner. 2015. “The Beveridge curve: A survey.” *Journal of Economic Literature* 53 (3): 571–630.

Faccini, Renato, and Eran Yashiv. 2022. “The importance of hiring frictions in business cycles.” *Quantitative Economics* 13 (3): 1101–1143.

Hosios, Arthur J. 1990. “On the efficiency of matching and related models of search and unemployment.” *The Review of Economic Studies* 57 (2): 279–298.

Hristov, Atanas, Christophe Planas, Werner Roeger, and Alessandro Rossi. 2017. “NAWRU Estimation Using Structural Labour Market Indicators.” European Economy Discussion Paper 069, Directorate General Economic and Financial Affairs (DG ECFIN), European Commission.

Kleven, Henrik J. 2021. “Sufficient statistics revisited.” *Annual Review of Economics* 13 515–538.

Manning, Alan. 2011. “Imperfect competition in the labor market.” In *Handbook of Labor Economics*, Volume 4. 973–1041, Elsevier.

Mas, Alexandre, and Amanda Pallais. 2019. “Labor supply and the value of non-work time: Experimental estimates from the field.” *American Economic Review: Insights* 1 (1): 111–26.

Michaillat, Pascal, and Emmanuel Saez. 2021. “Beveridgean unemployment gap.” *Journal of Public Economics Plus* 2 100009.

Michaillat, Pascal, and Emmanuel Saez. 2022. “$u^* = \sqrt{uv}$.” Working Paper 30211, National Bureau of Economic Research.

Mortensen, Dale T., and Christopher A. Pissarides. 1994. “Job creation and job destruction in the theory of unemployment.” *The Review of Economic Studies* 61 (3): 397–415.

Muehlemann, Samuel, and Harald Pfeifer. 2016. “The structure of hiring costs in

Germany: Evidence from firm-level data." *Industrial Relations: A Journal of Economy and Society* 55 (2): 193–218.

Muehlemann, Samuel, and Mirjam Strupler Leiser. 2018. "Hiring costs and labor market tightness." *Labour Economics* 52 122–131.

Pissarides, Christopher A. 2000. *Equilibrium unemployment theory.* MIT Press.

Shimer, Robert. 2005. "The cyclical behavior of equilibrium unemployment and vacancies." *American Economic Review* 95 (1): 25–49.